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VOL. 10 BATON ROUGE, LA., MAY, 1936 No. 8

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1. Through published standard papers on the culture aspects, humanism and history of mathematics to deepen and to widen public interest in its values.
2. To supply an additional medium for the publication of expository mathematical articles.
3. To promote more scientific methods of teaching mathematics.
4. To publish and to distribute to groups most interested high-class papers of research quality representing all mathematical fields.

Retrospect and Prospect

Twelve years have gone by since the college mathematics teachers of Mississippi and Louisiana, aided by nationally known figures in the field, became articulated with the Mathematical Association of America under the name Louisiana-Mississippi Section. Nine years ago, the Louisiana-Mississippi branch of the National Council of Teachers of Mathematics came into existence, being an organization of those interested primarily in secondary mathematics. For nine years these two bodies have pooled their annual programs, meeting at the same time and at the same place, one year in Louisiana, the next in Mississippi, with no variation.

The Mathematics News Letter, predecessor of the Magazine, did not spring into being full grown and armed as has been written of certain mythical creatures. Rather was it in the form of a puny infant that reckoned but little of possible future growth into a lusty adult. Speaking less figuratively, the News Letter made its initial appearance during the writer's first chairmanship of the Louisiana-Mississippi Section—a period antedating by a year the now historic beginning of the Louisiana-Mississippi Branch of N. C. T. M.

The idea of perpetuating what was begun as a mere leaflet was an after-thought, but the conception came within an hour after the historic beginning alluded to.

The story of the struggle to keep the mathematical home-fires burning by preserving the life of the little Journal can have no place on this page. Possibly such a story will never be written.

The years went by—years in which from every quarter of this country were revealed friends, sympathizers and helpers of the cause, those without whom our courage would have failed utterly. Foremost among these should be named H. E. Slaught, founder of the Mathematical Association of America.

In 1934, with the beginning of Volume 9 the name was changed to National Mathematics Magazine.

Mindful that the history of mathematics in America had shown that the 10th year is the fateful year of the privately promoted mathematical journal, that 9 or 10 years has been the maximum life period among those not surviving, we came to the close of the summer of 1935. Then fate fell on us gloriously! Our own institution, Louisiana State University, proposed generous terms of *adoption*!

With our acceptance of these terms, National Mathematics Magazine became one of its official publications. The conditions of the University's sponsorship were marked by sympathy for our years of financial sacrifice and by the desire of its President (James Monroe Smith) that Louisiana State University should share in the promotion of the science of sciences.

We have not space here to consider the Magazine's prospects. Those interested may prophesy.

S. T. SANDERS.

The First Printed Armenian Treatise on Geometry and Trigonometry

By A. A. SHAW
University of Arizona

NOTE—Father Lucas Derderiantz (c. 1830) was a member of the celebrated Armenian religious order, called the Mekhitharist Order, founded in September 8, 1701 by Abbot Mekhithar, an Armenian, hence the name of the Order. There are two branches of this Brotherhood: the first is in Venice, Italy, the second in Vienna, Austria. Each has its own schools and printing press, and has produced scholars in letters, arts and sciences, several of whom have secured international reputation.

Father Lucas' treatise, entitled "Elementary Mathematics" was published by the Mekhitharist press in Vienna in three volumes: the first volume was published in 1843 and is on elementary and advanced arithmetic and algebra (468 pages) and is described by the present writer in a paper shortly to be published in Venice. In this volume, while Father Lucas fully discusses various algebraic subjects, he constantly introduces and develops arithmetical topics, such as rules of divisibility of numbers, highest common factors, least common multiples, vulgar and decimal fractions including theory of recurring decimals, powers and roots, ratio and proportion, logarithm, etc. The reader feels that, while the author treats *algebra* quite fully, he makes it a handmaid to *arithmetic*—a commendable point indeed!

The second and third volumes appeared in 1846 and are on geometry and trigonometry as described below. The whole work is in ancient Armenian.

Father Lucas also published, by the same press, the following two works which were quite popular in his time among the Armenians: "Science of Commerce" in 1848, "Elements of Physics" in 1865.—A. A. S.

This is a continuation of the author's paper on "The First Printed Armenian Treatise on Arithmetic and Algebra" which dealt with the first volume of Father Lucas Derderiantz' mathematical treatise (1843). The present article deals with the second and third volumes of this treatise.

The second volume treats of plane and solid geometry and consists of an introduction and a table of contents, followed by 350 pages of geometric propositions and problems each treated in detail and in Euclidean style. In his arrangement of materials, the author does not follow Euclid's "Elements"—something which is rather remarkable and shows the independent spirit of Father Lucas, since in his time Euclid's text was the current text-book in England and in Europe in general. The volume begins with detailed discussions of the fundamental concepts, definitions, axioms, and postulates, and clearly shows the distinction of each. The text itself does not contain figures: all figures are numbered (203 altogether) and placed at the end of the volume in the form of four plates. The same method is used with regard to figures in the third volume. Throughout both volumes the author uses the two symbols for a right angle, i. e. R or $\pi/2$.

The plane geometry (201 pages) consists of seven chapters as follows: General Principles, On Congruences of Triangles and the truths established thereby, On Parallel and Proportional Lines and Similar Triangles, On Parallelograms and Polygons, On Properties of Circles, On Polygons Circumscribed about and Inscribed in a Circle, and On finding Plane Areas and concerning their Proportions and Ratios.

The solid geometry (147 pages) consists of four chapters as follows: On position of Straight Lines with respect to Plane Surfaces and of Planes with respect to one another, On Solid Angles, On Polyhedra (General Characteristics of Polyhedra, On Special Characteristics of Polyhedra, On Similarity of Polyhedra, on Mensuration of Polyhedra), On Regular Polyhedra, On Curved Bodies (Cylinder, Cone, and Sphere).

This list covers the contents of volume II. The whole work abounds in a large number of worked examples some of which are original and very interesting. They are treated both geometrically and analytically and quite fully. For example, while G. A. Wentworth allows only one page (page 350) to discussion of the five regular or Platonic solids in his text-book on geometry, Father Lucas devotes over seven pages to a detailed discussion of those solids and their several properties (pp. 306-13). With similar completeness are treated almost all the propositions and problems of the text and the reader will derive a good deal of pleasure and benefit by reading them.

Volume III, the smallest of the three, treats of plane trigonometry (pp. 1-76) and conic sections (pp. 77-120).

The trigonometry consists of two chapters. The first chapter (pp. 4-56) opens with a discussion of the general angle both in degrees and in radians. It then defines and illustrates the eight trigonometric functions in order and their inverse functions simultaneously—a very good method indeed. This is followed by a comparison of the trigonometric functions with one another and their calculations.

Father Lucas, however, does not define the trigonometric functions as ratios, *but as trigonometric lines*. For example, he defines sin A as follows: Consider an arc AP of a circle, radius CA (*not* necessarily unity). From P drop a perpendicular PM to CA. Then PM = sine of arc AP = sin A. And conversely, AP = $\angle$A = arc sin PM, i. e., AP or A is the arc or angle whose sine = PM.

Similarly are defined the rest of the seven functions and their inverses. These functions are, therefore, proportional to our trigonometric functions. Thus he gives (p. 14) $\sin 30° = \frac{1}{2}r$, $\cos 360° = r$ (p. 16), $\sin 45° = r\sqrt{2}/2$, $\sin 18° = \frac{2}{4}(-1+\sqrt{5})$, $\sin^2 A + \cos^2 A = r^2$

(p. 36), where r is the radius of the circle. But later on (p. 41 f) the author says: "For easy calculation and simplicity it is better to put "r = 1." His definitions of the functions when they become infinite are quite good and clear. For example, on p. 23 he says tan $90° = \tan \pi/2 = \infty$, i. e. a right angle has no tangent. $270° = 3\pi/2$ has no tangent, or tan $270° = \tan 3\pi/2 = \infty$ (p. 24). Again on p. 26 he remarks: 0° has has no cotangent or cot $0° = \infty$. With equal clarity he defines secants and cosecants when they become infinite (pp. 29, 30, 32.)

Chapter II (pp. 55-76) is given to the solutions of triangles, right-angled and general, discussing fully each case *theoretically*, but no numerical example is given with use of tables! But from p. 42 it appears that he made use of the following tables in his own classes:

Logarithmic Tables to seven places of decimals... by R. Shortrede. Edinburgh. 1884.

Callet François. Tables portatives de Logarithmes, etc. Edition stéréotype, gravée, fondue et impremée par Firmin-Didot. Paris, 1795.

Vega, Georg Freiherr, logarithmisch-trigomometrische Tafeln, nebst andern zum Gebrauch der Mathematick eingerichteten Tafeln und Formeln. 2 Bande. Leipzig, in der Weidman'schen Buchhandlung.

Shulz, J. Carl, neue erweiterte Sammlung logarithmischer und anderer zum Gebrauch der Mathematick undentbehrlicher Tafeln. 2 Bande. Berlin, 1778.

The last chapter of volume III deals with conic sections: parabola (pp. 79-91), ellipse (pp. 92-105), and hyperbola (pp. 106-120) treated geometrically and sometimes analytically. The chapter covers most of the fundamental propositions and constructions of conic sections, which are well done.

The whole text (three volumes) contains no exercises for the student. They are to be supplied by the teacher himself, a point for commendation as was emphasized in my first paper to which I referred.

At the ends of volumes I and III the author gives a good glossary where Armenian mathematical terms are translated into German and French, thus enabling the reader to have access to the mathematical literature in those two languages.

The entire work is a worthy contribution to mathematical literature and brings credit to the celebrated Mekhitarist Order of Vienna and Venice.

What precedes is in fact an abstract of an extended discussion of this work to be published later.

Circular Arc Figures

By LESTER R. FORD
Rice Institute

In the study of problems in the plane it is often advantageous to use complex variables for the designation of points. This is particularly true in the case of problems which involve circles and straight lines. The convenience of complex variables in this connection is not realized, the writer believes, by most engineers and by many mathematicians.

The usual rectangular coordinates (x,y) may be readily expressed in terms of z and its conjugate $\bar{z}$ from the relations

$$z=x+iy, \quad \bar{z}=x-iy. \tag{1}$$

We have

$$x=\frac{z+\bar{z}}{2}, \qquad y=\frac{i(\bar{z}-z)}{2}. \tag{2}$$

If these values be substituted into the equation of a curve in rectangular coordinates there results an equation in z and $\bar{z}$. Circles and straight lines have particularly simple equations. The circle with center a and radius r is

$$(z-a)(\bar{z}-\bar{a})=r^2.$$

This general equation of the circle may be written

$$A\bar{z}z+\bar{B}z+B\bar{z}+C=0, \tag{3}$$

where A (>0) and C are real. In this latter form the center is $-B/A$ and the radius is $\sqrt{B\bar{B}-AC}/A$. Equation (3) is the general equation of a straight line if $A=0$, $B\neq 0$.

These forms are especially useful when inversions in circles or reflections in lines are involved. It can be shown that the inverse of a point z in the circle (3) is the point z_1 defined by the equation

$$A\bar{z}_1+z\bar{B}z_1+B\bar{z}+C=0,$$

which is got from (3) by replacing z and z_1 and leaving $\bar{z}$ unaltered. The reflection of z in a line is similarly defined,

$$\bar{B}z_1+B\bar{z}+C=0.$$

The area S enclosed by a curve K is given by the wellknown line integral

$$S=\frac{1}{2}\int_K xdy-ydx.$$

Transforming this by means of equations (2), we have

$$S=\frac{i}{4}\int_K z\bar{dz}-\bar{z}dz.$$

The integral is taken counter-clockwise around K. It will be noted that $z\bar{dz}$ is the conjugate of $\bar{z}dz$; hence the formula may be written in the form

$$S=\frac{1}{2}I\left[\int_K \bar{z}dz\right], \tag{4}$$

where I indicates the imaginary part (that is, the coefficient of i) of the integral.

If K is made up of arcs of circles and pieces of straight lines the integrals involved are particularly simple. Thus along an arc of (3) we have

$$\bar{z}dz=\frac{-\bar{B}z-C}{Az+B}dz,$$

which is readily integrated. The straight line gives an even simpler integral. We find that if a piece of straight line K′ passes through the origin,

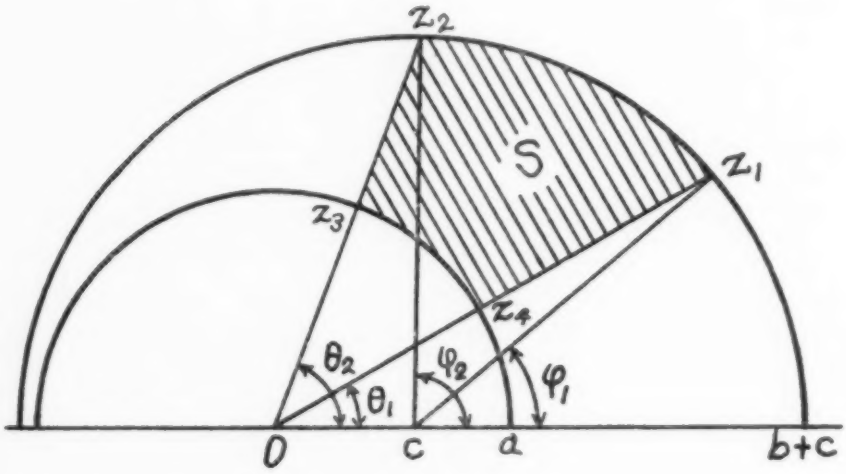
$$\bar{B}z+B\bar{z}=0,$$

the integral along it may be discarded; for we have

$$\int_{K'}\bar{z}dz=-\frac{\bar{B}}{B}\int_{K'}zdz=\left[-\frac{\bar{B}}{B}\frac{z^2}{2}\right]_{K'}=\left[\frac{\bar{z}z}{2}\right]_{K'},$$

which is real and so contributes nothing to the result.

We illustrate the use of Formula (4) by finding the area of the eccentric circular sector. This is a problem which evoked considerable discussion among the civil engineers two or three years ago.* The accompanying figure, in which the area to be found is shaded, is self explanatory.



The equations of the inner and outer circles are

$$\bar{z}z = a^2, \qquad (z-c)(\bar{z}-c) = b^2.$$

We discard the integrals along the straight lines z_2z_3 and z_4z_1. Integrating along the two circular boundaries of the region, we have

$$\int_{z_1}^{z_2} \bar{z}dz + \int_{z_3}^{z_1} \bar{z}dz = \int_{z_1}^{z_2} \left(c + \frac{b^2}{2-c} \right) dz - \int_{z_4}^{z_3} \frac{a^2}{a} dz$$

$$= [cz + b^2 \log(z-c)]_{z_1}^{z_2} - a^2[\log z]_{z_4}^{z_3}$$

The functions which appear in this result may be variously expressed. We put them in terms of the radii a, b, the distance c between the centers, and the angles which radii drawn to the corners of the region make with the line of centers. We have

$$z_1 = c + be^{i\varphi_1} = c + b(\cos\varphi_1 + i\sin\varphi_1),$$

$$z_2 = c + be^{i\varphi_2} = c + b(\cos\varphi_2 + i\sin\varphi_2),$$

*Civil Engineering, Vol. 3 (1933): M. N. Clair, p. 172; J. L. Nagle, p. 234; R. B. McWhorter, p. 284.

$$\log (z_1-c)=\log b+i\varphi_1,$$

$$\log (z_2-c)=\log b+i\varphi_2,$$

$$\log z_3=\log ae^{i\Theta_2}=\log a+i\Theta_2,$$

$$\log z_4=\log ae^{i\Theta_1}=\log a+i\Theta_1.$$

Substituting these values into the result of the integration and taking the imaginary part, we find that

$$2S=bc(\sin\varphi_2-\sin\varphi_1)+b^2(\varphi_2-\varphi_1)-a^2(\Theta_2-\Theta_1).$$

A Department of the History of Mathematics

Beginning with the October, 1936, issue of the Magazine—No. 1, Vol. 11—will be carried a new division of this Journal to be named,

HUMANISM AND HISTORY OF MATHEMATICS

The editor of the new Department will be Professor G. Waldo Dunnington. For several years Professor Dunnington has been engaged in writing a history of Karl Friedrich Gauss. This is the purpose of his year's stay in Germany, where, at the University of Göttingen, he is presently collaborating with scholars interested in a biography of the great German mathematician. Writing from the University of Göttingen, he says, "Probably the leading historian of mathematics is Professor Otto Neugebauer and I want to include for our inaugural paper (for the Magazine) in the fall an essay by him to bear the title '*History of Mathematics*'."

Integrating Odd Powers of Sec x

By W. VANN PARKER

1. In most text books on the calculus very little is said about the integral of odd powers of sec x when the power is greater than three. It is possible to obtain explicit formulas for the integral of any odd power of sec x and by these formulas the integral may be written out at once. I shall give two of these formulas in this note.

2. *Integration by partial fractions.* In the following n is a positive integer. We may write.

$$\int \sec^{2n-1} u\, du = \int \frac{du}{\cos^{2n-1} u} = \int \frac{\cos u\, du}{(1-\sin^2 u)^n}.$$

If we let $\sin u = v$, this integral becomes

$$\int \frac{dv}{(1-v^2)^n}.$$

We may resolve the integrand into partial fractions as

$$\frac{1}{(1-v^2)^n} \equiv \frac{A_1}{1-v} + \frac{A_2}{(1-v)^2} + \cdots + \frac{A_n}{(1-v)^n} + \frac{B_1}{1+v} + \frac{B_2}{(1+v)^2} + \cdots + \frac{B_n}{(1+v)^n}$$

It is evident that $B_i = A_i$, since this is an even function of v. Write $1+v=x$ and $1-v=y$ and $f(x) = A_1 x^{n-1} + A_2 x^{n-2} + \cdots + A_n$. The equation for determining the A's then becomes

$$x^n f(y) + y^n f(x) = 1, \text{ where } x+y=2.$$

Divide by x^n and we have

$$f(y) + y^n \left[\frac{A_1}{x} + \frac{A_2}{x^2} + \cdots + \frac{A_n}{x^n} \right] = \frac{1}{x^n}$$

If we differentiate k times with respect to x, $(k<n)$, we get

$$f^{(k)}(y)(-1)^k + \text{terms multiplied by } y \equiv \frac{(-1)^k(n+k-1)!}{(n-1)!\ x^{n+k}}.$$

Let $y=0$ and hence $x=2$ and we get

$$(-1)^k k!\, A_{n-k} = \frac{(-1)^k(n+k-1)!}{(n-1)!\ 2^{n+k}},\quad k=0,1,2,\ldots,(n-1).$$

If we set $n-k=i$, we have

$$A_i = \frac{{}_{2n-i-1}C_{n-i}}{2^{2n-i}}.$$

If we substitute these values and integrate term by term, we get,

$$\int \sec^{2n-1}u\,du = \frac{{}_{2n-2}C_{n-1}}{2^{2n-1}} \log\left(\frac{1+\sin u}{1-\sin u}\right)$$

$$+\sum_{i=2}^{n} \frac{{}_{2n-i-1}C_{n-i}}{(i-1)2^{2n-i}}\left[\frac{1}{(1-\sin u)^{i-1}} - \frac{1}{(1+\sin u)^{i-1}}\right]+C.$$

3. *Integration by parts.* If we use the usual method of integrating by parts, we get the reduction formula

$$\int \sec^{2n+1}x\,dx = \frac{\sec^{2n-1}x\tan x}{2n} + \frac{2n-1}{2n}\int \sec^{2n-1}x\,dx + C.$$

If we proceed by induction, we get

$$\int \sec^{2n+1}x\,dx = \frac{{}_{2n}C_n}{4^n}\left[\log|\sec x+\tan x| + \sum_{i=1}^{n}\frac{1}{2i}\,\frac{4^i}{{}_{2i}C_i}\sec^{2i-1}x\tan x\right]+C.$$

This formula enables us to write out the integral of any odd power of sec x at once. For example

$$\int \sec^{11} x\,dx = {}^1/_2 \cdot {}^3/_4 \cdot {}^5/_6 \cdot {}^7/_8 \cdot {}^9/_{10} \log|\sec x + \tan x|$$
$$+ {}^1/_2 \cdot {}^3/_4 \cdot {}^5/_6 \cdot {}^7/_8 \cdot {}^9/_{10} \sec x \tan x + {}^1/_4 \cdot {}^5/_6 \cdot {}^7/_8 \cdot {}^9/_{10} \sec^3 x \tan x$$
$$+ {}^1/_6 \cdot {}^7/_8 \cdot {}^9/_{10} \sec^5 x \tan x + {}^1/_8 \cdot {}^9/_{10} \sec^7 x \tan x + {}^1/_{10} \sec^9 x \tan x + C$$

The law of formation of the coefficients, from the integers 1,2,3,. . .$2n$, is evident.

It is interesting to note in this connection that the sum of these coefficients is unity for all values of n. This may be shown by evaluating the indeterminate form

$$\lim_{x \to 0} \frac{\int_0^x \sec^{2n+1} y\,dy}{\tan x}$$

This result gives us the following relation

$$1 + \sum_{i=1}^{n} \frac{1}{2i} \cdot \frac{4^i}{{}_{2i}C_i} = \frac{4^n}{{}_{2n}C_n}.$$

The Covariants of a Quadric Surface

By H. LYLE SMITH

In books on solid analytics it is shown that if the quadratic expression

$$C_0(xyz) = a_{11}x^2 + 2a_{12}xy + 2a_{13}xz + 2a_{14}x + a_{22}y^2 + 2a_{23}yz + 2a_{24}y + a_{33}z^2 + 2a_{34}z + a_{44} \tag{1}$$

is subjected to an Euclidean linear transformation, then the determinants

$$\begin{vmatrix} a_{11}-\lambda, & a_{12}, & a_{13} \\ a_{21}, & a_{22}-\lambda, & a_{23} \\ a_{31}, & a_{32}, & a_{33}-\lambda \end{vmatrix} \equiv -\lambda^3 + I_1\lambda^2 - I_2\lambda + I_3 \tag{2}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} \equiv I_4 \tag{3}$$

are absolutely invariant; in the case of (2) this is true for every value of λ so that I_1, I_2, I_3 are individually invariant. The four invariants $I_1, \ldots, I_4$ are applied to the problem of determining the coefficients of the various canonical forms of (1). They do not however suffice to give a complete solution of this problem and thus L. J. Paradiso*, was led to consider the covariants of $C_0(xyz)$. He obtained these by means of the Lie theory of continuous groups—a method hardly appropriate to an elementary course in solid geometry. In 1927 Franklin† gave a direct treatment of Paradiso's covariants, making use of matrix theory and planar coordinates. In seeking a still more elementary treatment in terms of matrices alone, the writer was led in April, 1935 to a different set of covariants. This set seems to be particularly simple in theory and convenient in practice.

*Paradiso, A classification of second degree loci of space. American Mathematical Monthly, vol. 33 (1926), pp. 406-418.

†Philip Franklin, American Mathematical Monthly, vol. 34, pp. 453-467.

§1. *A covariant of two quadrics.* Let $f(x,y,z)$, $g(x,y,z)$ be given by

$$f(xyz) = \xi A \xi_0 \tag{4}$$

$$g(xyz) = \xi B \xi_0 \tag{5}$$

where

$$\xi = (xyz1) \tag{6}$$

$$A = (a_{ij} | i,j = 1,2,3,4) \tag{7}$$

$$B = (b_{ij} | i,j = 1,2,3,4) \tag{8}$$

and where the subscript 0 indicates that the affected matrix is to be transposed.

Now let T denote a matrix of the form

$$T = \begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 \\ c_{21} & c_{22} & c_{23} & 0 \\ c_{31} & c_{32} & c_{33} & 0 \\ x_0 & y_0 & z_0 & 1 \end{pmatrix} \tag{9}$$

where the sub-matrix made up of the c's is an orthogonal one. Then either of the equations

$$\xi = \xi' T, \quad \xi_0 = T_0 \xi'_0 \tag{10}$$

will represent the most general Euclidean transformation.

Under (10), $f(xyz)$, $g(xyz)$ become

$$f'(x'y'z') = \xi' A' \xi'_0, \quad g'(x'y'z') = \xi' B' \xi'. \tag{11}$$

where

$$A' = TAT_0, \quad B' = TBT_0 \tag{12}$$

Now consider the quadratic function

$$h(xyz) = \xi(AJB)\xi_0 \tag{13}$$

where

$$J = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{14}$$

The reader will easily verify that

$$T_0 J T = J. \tag{15}$$

If we now define $h'(x'y'z')$ to be $\xi'(A'JB')\xi'_0$, we have

$$\begin{aligned} h'(x'y'z') &= \xi'(A'JB')\xi'_0 \\ &= \xi' TAT_0JTBT_0\xi'_0 \qquad \text{(by (12))} \\ &= \xi' TAJBT_0\xi_0' \qquad \text{(by (15))} \\ &= \xi AJB\xi_0 \qquad \text{(by (10))} \\ &= h(xyz). \end{aligned}$$

We have thus shown that $h(xyz)$ is a covariant of $f(xyz)$ and $g(xyz)$.

§2 *Another formula for the covariant* $h(xyz)$. We assume the matrices A,B to be symmetric, that is, that

$$A = A_0, \quad B = B_0; \tag{16}$$

and also introduce the functions

$$\begin{aligned} \tfrac{1}{2}f_1 &= a_{11}x + a_{12}y + a_{13}z + a_{14} \\ \tfrac{1}{2}f_2 &= a_{21}x + a_{22}y + a_{23}z + a_{24} \\ \tfrac{1}{2}f_3 &= a_{31}x + a_{32}y + a_{33}z + a_{34} \end{aligned} \tag{17}$$

with similarly defined expressions $\frac{1}{2}g_1$, $\frac{1}{2}g_2$, $\frac{1}{2}g_3$. It is easily verified that

$$(\tfrac{1}{2}f_1, \tfrac{1}{2}f_2, \tfrac{1}{2}f_3, 0) = \xi A_0 J \tag{18}$$

$$(\tfrac{1}{2}g_1, \tfrac{1}{2}g_2, \tfrac{1}{2}g_3\ 0) = \xi B_0 J \tag{19}$$

On multiplying (18) by (19) transposed, we get

$$\begin{aligned} \tfrac{1}{4}(f_1g_1 + f_2g_2 + f_3g_3) &= \xi A_0 JJB\xi_0 \\ &= \xi AJB\xi_0 \end{aligned}$$

that is,

$$h(xyz) = \tfrac{1}{4}(f_1g_1 + f_2g_2 + f_3g_3), \tag{20}$$

which is a convenient formula for $h(xyz)$.

The reader will easily verify that if c_{ij} is the element in the i-th row and j-th column of AJB, then

$$c_{ij} = \Sigma a_{ik}b_{kj} \tag{21}$$

where the summation is for values of k from 1 to 3 inclusive.

§3. *Complete system of invariants and covariants of a quadric surface.* We saw in §1 how to build up a covariant of two quadratic expressions. If we apply this process to $C_0(xyz)$ and itself we obtain the covariant

$$C_1(xyz) = \xi AJA\xi_0. \tag{22}$$

If we apply the process to $c_0(xyz)$ and $c_1(xyz)$ we obtain the covariant

$$C_2(xyz) = \xi AJAJA\xi_0. \tag{23}$$

We may continue this process indefinitely, but all further covariants obtained are expressible in terms of C_0, C_1, C_2 and I_1, I_2, I_3, I_4 which thus form a complete system. This can be shown directly. But the completness of the system follows from the fact shown later that the above system completely determines all geometric properties of the quadric $C_0(xyz) = 0$.

§4. *Characterization of the canonical forms.* We assume that the reader knows that the quadric $C_0(xyz) = 0$ can be reduced by an Euclidean transformation to one of the canonical forms

(I)	$C_0'(x'y'z') = a'_{11}x'^2 + a'_{22}y'^2 + a'_{33}z'^2 + a'_{44} = 0$	$(a'_{11} \cdot a'_{22} \cdot a'_{33} \neq 0)$
(II)	$C_0'(x'y'z') = a'_{11}x'^2 + a'_{22}y'^2 + 2a'_{34}z' = 0$	$(a'_{11}a'_{22}a'_{34} \neq 0)$
(III)	$C_0'(x'y'z') = a'_{11}x'^2 + a'_{22}y'^2 + a'_{44} = 0$	$(a'_{11}a'_{22} \neq 0)$
(IV)	$C_0'(x'y'z') = a'_{11}x'^2 + 2a'_{24}y' = 0$	$(a'_{11}a'_{24} \neq 0)$
(V)	$C_0'(x'y'z') = a'_{11}x'^2 + a'_{44} = 0$	$(a'_{11} \neq 0)$

We leave it to the reader to show that these cases are characterized by the conditions

(I)	$I_3 \neq 0$
(II)	$I_3 = 0,\ I_2 \neq 0,\ I_4 \neq 0$
(III)	$I_3 = 0,\ I_2 \neq 0,\ I_4 = 0$
(IV)	$I_3 = I_2 = 0,\ C_2 - I_1C_1 = \text{constant} \neq 0.$
(V)	$I_3 = I_2 = 0,\ C_2 - I_1C_1 = 0.$

The covariants are needed only to distinguish cases IV, V from each other and the other cases.

§5. *Detailed treatment of case I.* Let us assume that $I_3 \neq 0$ so that we have case I. On equating (2) for C_0 and C'_0 we get

$$(\lambda - a'_{11})(\lambda - a'_{22})(\lambda - a'_{33}) = \lambda^3 - I_1\lambda^3 + I_2\lambda - I_3 \tag{24}$$

so that a'_{11}, a'_{22}, a'_{33} can be found by solving a cubic equation with known coefficients. On equating I_4 and I'_4 we get (25) $I_3a'_{44} = I_4$ which determines a'_{44}. Thus all the coefficients of the canonical form are determined by the invariants. It remains to find x', y', z' in terms of x,y,z. To this end we equate C'_0, C'_1, C'_2 respectively to C_0, C_1, C_2. The result is

$$\begin{aligned} &a'_{11}x'^2 + a'_{22}y'^2 + a'_{33}z'^2 = C_0 - I_4/I_3. \\ &a'^2_{11}x'^2 + a'^2_{22}y'^2 + a'^2_{33}z'^2 = C_1 \\ &a'^3_{11}x'^2 + a'^3_{22}y'^2 + a'^3_{33}z'^2 = C_2. \end{aligned} \tag{26}$$

Suppose first that a'_{11}, a'_{22}, a'_{33} are distinct. Then (26) can be solved uniquely for x'^2, y'^2, z'^2. For, the determinant of the left member of (26) reduces to $I_3(a'_{33}-a'_{22})(a'_{33}-a'_{11})(a'_{22}-a'_{11})$, which is not zero. Suppose $a'_{11}=a'_{22}$, $a'_{22}\neq a'_{33}$. In this case the first two equations of (26) reduce to

$$a'_{11}(x'^2+y'^2)+a'_{33}z'^2=C_0-I_4/I_3 \tag{27}$$
$$a'^2_{11}(x'^2+y'^2)+a'^2_{33}z'^2=C_1$$

which may be solved for $x'^2+y'^2$ and z'^2 since the determinant of the left member is $a'_{11}a'_{33}(a'_{33}-a'_{11})$, which is not zero. The remaining case is that in which $a'_{11}=a'_{22}=a'_{33}$. In this case we find

$$x'^2+y'^2+z'^2=(C_0-I_4/I_3)/a'_{11} \tag{28}$$

and the locus is a sphere. We thus see that in case I, the invariants and covariants suffice to determine all geometric properties of the quadric. We leave the detailed treatment of the other cases to the reader.

§6. *Another formula for the covariant* $h(xyz)$. We have given above a purely algebraic treatment of the covariants of a quadric surface. The theory given is easily adapted to use in a book on solid analytics. We remark here that the covariants used are easily obtained by vector analysis. For the functions $f(xyz)$, $g(xyz)$ of §1 may be thought of as defining two scalar fields in space. We then have

$$\text{grad } f=f_1i+f_2j+f_3k \tag{29}$$
$$\text{grad } g=g_1i+g_2j+g_3k.$$

Hence

$$h=\tfrac{1}{4}\text{ grad } f\cdot\text{grad } g. \tag{30}$$

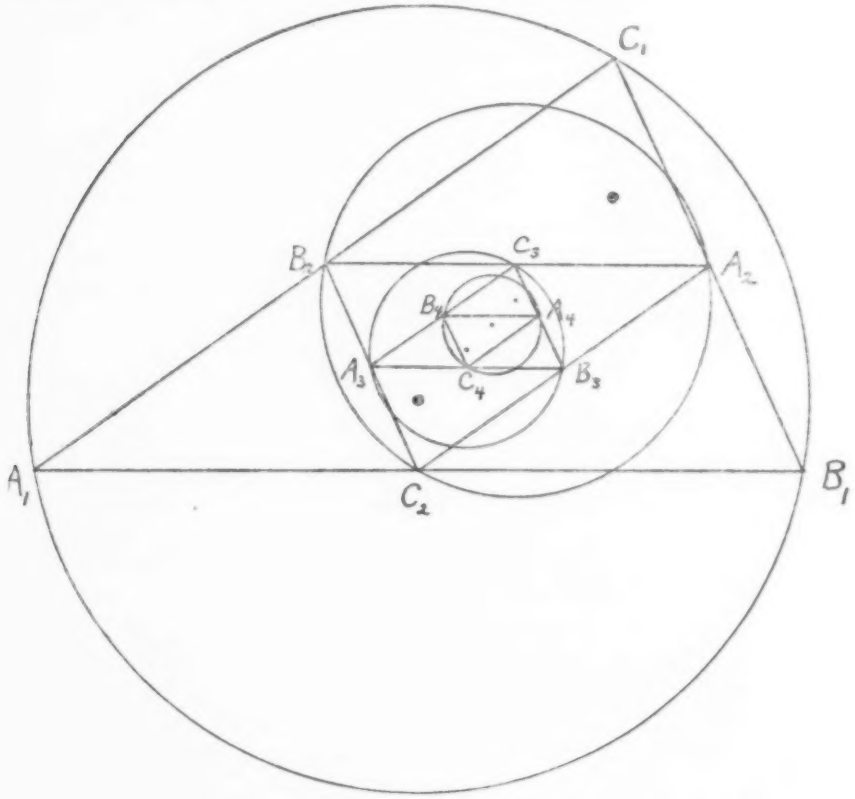
§7. *Relation to Paradiso's system.* The reader familiar with Paradiso's paper will find that his system of covariants is

$$c_0,\ I_1c_1-c_2,\ I_3c_1-I_1c_2+c_3.$$

On a System of Circles

By KARLETON W. CRAIN
Purdue University

In the triangle $A_1B_1C_1$, let the midpoints of the sides determine the similar triangle $A_2B_2C_2$. Let the midpoints of the sides of triangle $A_2B_2C_2$ determine the similar triangle $A_3B_3C_3$ and so ad infinitum. Now draw the circumcircles of all these triangles. Since the circumcircle, Υ_i, of triangle $A_iB_iC_i$ is the Nine Point Circle of the triangle $A_{i-1}B_{i-1}C_{i-1}$, its radius will be one-half the radius of Υ_{i-1}.



Now if we take the radius of Υ_1 as unity, the sum of the areas of this infinite system of circumcircles gives the series $\pi(1+1/4+1/16+\ldots)$ $=4\pi/3$. In other words, the sum of the areas of Υ_i $(i=2,3,4,\ldots)$ is

equal to one-third the area of Υ_1. Furthermore, the sum of the circumferences of this infinite system of circumcircles gives the series $2\pi(1+1/2+1/4+\ldots.)=4\pi$. The sum of the circumferences of $\Upsilon_i(i=2,3,4,\ldots.)$ is equal to the circumference of Υ_1.

Let O_i and H_i be the circumcenter and orthocenter of triangle $A_iB_iC_i$. Since O_{i+1} is the midpoint of the segment O_iH_i, and since the point O_i is identical with the point H_{i+1} all these centers of these circles lie on the Euler line of the triangle $A_1B_1C_1$.

The center of Υ_i as $i\to\infty$ approaches a particular point, the centroid of triangle $A_1B_1C_1$. For if we take O_1 as the origin and H_1 as the point 2 on the Euler line as an exis, then as O_{i+1} is one-half of the distance from O_i to H_i, the limiting point is located by summing the series $1-1/2+1/4-1/8+\ldots.$ The sum is $2/3$. Therefore, the limiting position required is the centroid as it is the point on the Euler line which is one-third of the distance from O_1 to H_1.

Foreign Exchanges

Mathematical publications received in exchange for the National Mathematics Magazine:

Matematisk Institut (Det Kongelige Frederiks Universitet, Olso, Norvege)—*Norsk Matematisk Tidsskrift.*

Matematischen Seminar der Hamburgischen Universitat—*Abhandlungen.*

Seminario Matematico, R. Universita degli Studi (Roma, Italia)—*Rendiconti.*

Kungl. Universitetets Bibliotek (Uppsala, Sweden)—Publications.

Mathematical Gazette (London, England).

Boletin Matematico (Buenos Aires, Argetina).

Pitagoras (Mexico City, Mexico).

Revista Matematica Hispano Americana (El Administrador, Sociedad Matematica Espanola, Duque de Medinaceli, 4, Madrid, Spain).

Matematisk Tidsskrift (Universitetets Matematiske Institut, Blegdamsvej 15, Copenhagen, Denmark.)

Bulletin of the Calcutta Mathematical Society (Calcutta Mathematical Society, 92 Upper Circular Road, Calcutta.)

Mathematisches Seminar Fredirich-Wilhelms Universitat, Berlin, Germany—*Schriften des Mathematischen Seminars.*

The Teacher's Department

Edited by
JOSEPH SEIDLIN

The Place of Mathematics and its Teaching in the Schools of This Country

BY THE READERS OF COLLEGE MATHEMATICS

Toward the end of February and through the first week of March I sent out 1000 letters, a copy of which is reproduced below, to teachers of college mathematics in 520 colleges or universities.

At the St. Louis meeting of the several Mathematics Associations there appeared to be—both in the program and in "asides"—an unusual amount of discussion with reference to the apparently precarious status of mathematics in the curricula of the schools of this country.

Whatever studies have been made seem to indicate that as a required subject mathematics is losing ground both in secondary schools and in colleges.

Many of those most perturbed by the evident trend are of the opinion that teachers of mathematics are unanimous, or nearly so, in their desire (and belief) that mathematics *should be required of most, if not all, pupils of the ninth and tenth grades.*

Personally, I am not certain that that is so. I have decided to replace guesses and opinions by more or less reliable statistical evidence. I am, therefore, addressing the following set of questions to 1,000 of our colleagues chosen as nearly "random" as I know how.

Please feel free to modify any of the questions if you think that in the given form the question is not amenable to a "Yes" or "No" answer. Since the results and the analysis of the replies to these questions are to appear in the May issue of the National Mathematics Magazine, would you be good enough to fill out and return the questionnaire at your earliest convenience.

Sincerely yours,
JOSEPH SEIDLIN.

I. Do you believe that at least one year of algebra and one year of geometry should be required of all high school pupils:
   (a) As now taught?.....[ ]
   (b) Changed (presumably improved) in content?.....[ ]

II. Do you believe that at least six semester hours of mathematics should be required of all students in a liberal arts college?.....[ ]

III. Do you believe that mathematics at the level indicated below, requires "special mathematical ability?"
   (a) Ninth grade.....[ ]
   (b) Tenth grade.....[ ]
   (c) First year college.....[ ]

IV. Do you believe that only "superior" or "scientifically-minded" students should be encouraged to elect mathematics beyond the 8th, 9th, 10th, 11th, 12th, 13th (first year college) grade? Indicate grade.....[ ]

V. Comments:

To date, April 21st, I have received 764 replies. Three letters were returned by colleagues of the men who died since February 20th; four letters came back because of faulty address or "change of address"; many letters must have been lost in some of the mails lost or misplaced during the period of the most recent floods. In the last two days only one letter arrived. Quite possibly others will arrive too late to be included in this tabulation.

TABULATION OF REPLIES

	YES		NO		DOUBTFUL		BLANK			
I. Do you believe that at least one year of algebra and one year of geometry should be required of all high school pupils:		%		%		%		%		%
(a) As now taught?	270	*35*	206	*27*	32	*4*	256	*34*		
(b) Changed...in content?	452	*59*	96	*13*	54	*7*	162	*21*		
II. Do you believe that at least six semester hours of mathematics should be required of all students in a liberal arts college?	308	*40+*	376	*49+*	55	*7+*	25	*3+*		
III. Do you believe that mathematics at the level indicated below, requires "special mathematical ability?										
(a) Ninth grade	34	*4*	668	*88*	22	*3*	40	*5*		
(b) Tenth grade	39	*5*	633	*83*	30	*4*	62	*8*		
(c) First year college	94	*12*	557	*73*	50	*7*	63	*8*		

IV. Do you believe that only "superior" or "scientifically minded" students should be encouraged to elect mathematics beyond the... Indicate grade.	8th grade	9th grade	10th grade	11th grade	12th grade	First Year college	Second Year college	"Yes"	"No"	Blank
	13	22	45	52	61	283	31	37	184	36
%	1.7	2.9	5.9	6.8	8	37	4	4.8	24	4.7

It would be presumptuous of me to thank our colleagues for this record-breaking response to a circular letter. But I do appreciate the many friendly and encouraging letters and briefer comments crediting me with perhaps more than I deserve for launching these queries.

An unexpected development is the volume and nature of the "comments", for which I provided perfunctorily so little space in the

questionnaire. I must confess that item V was included mainly as a matter of form and as a token of undue optimism. However, it is not the first time in the history of research that a "by-product" or an "after-thought" turns out to be a matter of great significance. The Teacher's Department of the October and November issues of the Magazine will be devoted chiefly to the report, analyses, and summaries of the comments.

ANALYSIS

The replies to I(a) indicate definitely that teachers of college mathematics are a "house almost evenly divided" in the matter of requiring two years of high school mathematics, as now taught, of all high school pupils. Irrespective of consequences, the position held by some of our colleagues that "teachers of mathematics are unanimous in their belief that mathematics should be required of all pupils of the ninth and tenth grades" is no longer tenable.

The replies to I(b) indicate emphatically that teachers of college mathematics believe that "all is not well" with the traditional content and, as will appear later, in the analysis of the "comments", the teaching of the first two years of high school mathematics. Nearly 60% of those replying believe that two years (or more) of the "right kind of mathematics", "properly taught", should be required of all high school pupils.

The categorical replies to II will not prove very helpful to the revisionists of the liberal arts college curricula. Nearly half of our colleagues believe that first year college mathematics is not essential to a "liberal education". And so, the 40% who believe that a year of college mathematics *is* essential to a "liberal education" must convert to their belief not only the educators outside the field of mathematics but, also, half of their brethren within.

The replies to III are fairly conclusive. It is nearly the unanimous belief of our colleagues that elementary mathematics of the high school and junior college levels do NOT require "special mathematical ability". I wish it were possible to broadcast this "fact" to all pupils and teachers and parents. I suppose all teachers of mathematics have shared the confidences of many of their students, who claimed a positive inheritance of a negative ability in mathematics. It seems to me that the teaching and learning of mathematics, as well as its status in the curricula of our schools, would be aided by the destruction of the myth of "special mathematical ability".

Question IV was intended to determine the degree of exclusiveness of mathematics cherished by the teachers of the subject. Should

the study of mathematics beyond a given level be limited to a "select" group? If teachers of mathematics had their way, would they permit or encourage the "casting of pearls before swine"? It is rather significant that 24% of our colleagues would discourage no one from "drinking at the fountain". On the other hand, almost an equal number of our colleagues would set up an aristocracy of ability or attitude for students of mathematics beyond the high school level. And some (2%) would encourage only the elect to continue the study of mathematics beyond the eighth grade. Most of those in the "blank" column commented on the vagueness of the terms "superior" or "scientifically-minded".

What has the questionnaire accomplished? Aside from establishing a "professional" stand on matters about which much doubt and curiosity has been expresssed both in and outside the profession; aside from representative opinion which cannot be fairly or honestly disregarded or minimized by educationists and other curriculum builders; aside from illuminating and intrinsically-instructive comments contributed by many of the leading spirits of the profession; there is the mere volume of "returns" proclaiming in a convincing manner the earnest and intense professional interest of our group. Perhaps, some will say, it is an interest only recently acquired. What of it! It is here to encourage to still greater efforts the real teachers and friends of mathematics as well as to give pause to the misinformed and misguided fadists in the field of education. May the sincere and better informed general educators put to proper use the above data and the forthcoming analysis of authoritative and pertinent comments on the rôle of mathematics in genuine education.

Mathematical Notes

Edited by
L. J. ADAMS and I. MAIZLISH

Professor W. E. Byerly, professor emeritus of mathematics at Harvard University, died on December 20, 1935, at the age of 87 years. In 1873 Professor Byerly received the first doctorate in mathematics awarded at Harvard University.

A meeting of the American Mathematical Society is scheduled for June 18 at Seattle, Washington.

The American Association of University Professors maintains an Appointment Service which is open to members only. Listings of professors available, with subject and brief description of training, are in the Bulletin of the Association.

Research workers in mathematics often find material of interest and outlet for articles in the Arkiv för Matematik, Astronomie och Fysik. This journal is published by Almquist and Wiksells Boktryckeri, Stockholm. Articles in French, German, English and the Scandinavian languages are included.

Professor Jacques Hadamard's Jubilee is being celebrated by the publication of a volume of his more important papers and by special meetings of mathematical societies the world over.

Dr. Earle Raymond Hedrick, chairman of the mathematics department of the University of California at Los Angeles, will present two summer school courses at the School of Education of New York University. The dates are July 6-August 14, and the subjects are the teaching of senior high school mathematics and the teaching of college mathematics.

A real contribution to the literature on *The Meaning of Probability* is the article of that title in the March number of the Journal of the American Statistical Association.

The program of the April 2nd meeting of the Ohio Section of the Mathematical Association of America was scheduled to include the following addresses:

1. *Solutions of Systems of linear differential equations in the vicinity of singular points.* Professor Jesse Pierce.

2. *Apportionment of representatives.* Professor H. A. Bender.
3. *Getting the best from our students.* Professor O. L. Dustheimer.
4. *Calculation of potential field about a wire ring and about a flat plate.* Professor G. W. Spenceley.
5. *A remark on the function log x.* Professor Tibor Rado.
6. *The application of determinants and projective transformations to alignment charts.* Professor O. E. Brown.
7. *The presentation of quantum mechanics from an elementary standpoint.* Professor C. C. Torrance.
8. *An experiment in cooperative teaching.* Professor J. H. Weaver.
9. *What the mathematician explains.* Professor Mary Sinclair.
10. *Geometric proofs of multiple angle theorems.* Professor Wayne Dancer.

Professor Jesse Pierce is chairman of the Ohio section and Professor J. B. Brandeberry is chairman of the program committee.

The fourteenth annual meeting of the Southeastern section of the Mathematical Association of America was held at the University of South Carolina on April 17-18. Dr. Walter Bartky of the University of Chicago was guest speaker. His addresses were "Expanding Universe—Pro and Con", and "Systems of Linear Differential Equations." Some twenty-two papers were read by members of the Association. The three meetings were presided over by Professor F. W. Kokomoor of the University of Florida and Professor J. B. Coleman of the University of South Carolina.

In addition to the address on *Mathematics and Social Phenomena* by Dr. Forrest Ray Moulton, invited speaker, the Indiana section of the Mathematical Association of America included on its program the following papers:

1. *Some examples in mathematics.* Professor Herbert A. Meyer.
2. *An investigation of attitudes of high school students toward mathematics.* Miss Helen Darley, Manchester College.
3. *Arithmetic three hundred years ago and now.* Professor T. E. Mason, Purdue University.
4. *Mascheroni's Geometry of the Compass.* Professor L. S. Shively, Ball State Teachers College.
5. *Note on Bierbach's trisection of an angle.* Mr. Edwin L. Godfrey, Indiana University.

6. *A simple recursion method for solving a system of linear equations.* Professor C. Lanczos, Purdue University.
7. *Synthetic treatment of cycloids.* Professor J. H. Butchart, Butler University.
8. *A system of equations connected with a comet orbit.* Professor K. P. Williams, Indiana University.
9. *A problem in infinite quadratic forms.* Mr. R. L. Deputy, Indiana University.

The meeting of the Indiana section was held May 1 and 2 at Manchester College.

The Indiana section sponsors a mathematics contest.

The first preliminary announcement of the forty-second summer meeting and nineteenth colloquium of the American Mathematical Society indicates that the occasion will be of unusual importance and benefit to American mathematicians. The following scholars have been invited as lecturers: Professors Elie Cartan of the University of Paris, L. E. Dickson of the University of Chicago, R. A. Fisher of the University of London, G. H. Hardy of the University of Cambridge, and Tullio Levi-Civita of the University of Rome. A joint session with the American Astronomical Society will be held with Professor A. S. Eddington of the University of Cambridge and Professor Levi-Civita as principal speakers. Professor G. D. Birkhoff of Harvard is chairman of the committee on arrangements. The dates are August 31-September 5.

The joint annual conventions of the Mississippi Academy of Science, the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics and the Louisiana-Mississippi section of the Mathematical Association of America were held at Hattiesburg, Mississippi on March 13th and 14th. The meeting of the Mathematical Association of America was presided over by Dr. H. L. Smith of Louisiana State University. The following papers were presented:

1. *True Probable Errors.* Professor George A. Baker, Mississippi Woman's College.
2. *A Discussion of Teaching Problems, with Especial Reference to the Superior Student.* Professor R. L. O'Quinn, Louisiana State University.
3. *A Geometric Definition of the Torsion of a Skew Curve.* Professor H. L. Smith, Louisiana State University.

4. *Real and Complex Roots of an Algebraic Equation by the Method of Graeffe.* Professor J. E. Thompson, Tulane University.
5. *A Topic on Cyclic Groups of General Linear Fractional Transformations.* Professor T. A. Bickerstaff, University of Mississippi.
6. *An Evaluation and Comparison of Objective and Subjective Tests in Mathematics.* Professor J. O. Hassler, University of Oklahoma.

The presiding officer of the meetings of the Mississippi Academy of Science was Professor O. V. Austin. Some eleven scientific papers were read.

The meeting of the National Council of Teachers of Mathematics was presided over by Miss Loreen Dyson, Bolton High School, Alexandria, Louisiana. The program of this meeting included:

1. *Report of Committee on High School and College Mathematics.* Dr. P. K. Smith, Louisiana Polytechnic Institute.
2. *Tricks in Finance.* Dr. Irby C. Nichols, Louisiana State University.
3. *A Mathematics Teacher Faces the Future.* Miss Virginia Felder, Copiah-Lincoln Junior College.

One of the largest publishers of mathematical works in France is the firm of Gauthiers-Villars, located at 55, Quai des Grands-Augustins, in Paris. Their *catalogue général* lists many important books, papers and mathematical journals.

The Bulletin de la Société Mathématique de Grèce is one of the not so widely known mathematical journals. It is published in Greek, although some of the articles are in French and German. It is devoted to pure research, activities of the Société, and the philosophy and teaching of mathematics. The president of the Société is Mr. N. Sakellariou, Rue Voulis 3, Athenes.

L. J. ADAMS.

Problem Department

Edited by
T. A. BICKERSTAFF

This department aims to provide problems of varying degrees of difficulty which will interest anyone who is engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and to solve problems here proposed.

Problems and solutions will be credited to their authors.

While it is our aim to publish problems of greatest interest to readers, it is believed that regular text-book problems are, as a rule, less interesting than others. Therefore, other problems will be given preference when the space for problems is limited.

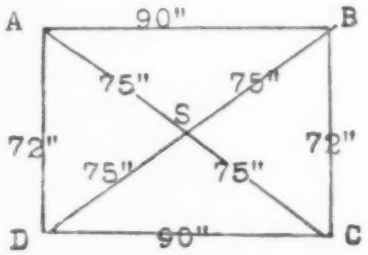
Send all communications about problems to T. A. Bickerstaff, University, Mississippi.

SOLUTIONS

No. 82. Proposed by Dewey C. Duncan, University of California.

A spider is resting at the top of a pyramid whose base is a rectangle $7\frac{1}{2}$ by 6 ft. The top of the pyramid is four feet above the center of the base. The spider wishes to make a complete journey in the shortest possible way, traversing each edge of the pyramid at least once and ending his course at top. How long is his journey? In how many ways can it be made?

Solved by *A. C. Briggs.*



The base is a rectangle, 90″x72″, and the distance from the top of the pyramid to each corner of the base is 75″. The total length of the 8 edges = 75″x4 + 90″x2 + 72″x2 = 624″ = 52′. When necessary

to travel an edge the second time, choose the shortest edge (width of base = 72″).

Let the spider at S descend to A and travel the entire perimeter of the base via B, C, D to A, and then return to D (traveling AD the second time). From D ascend to S and descend to B. Traverse BC the second time to C and ascend to S, traversing the 8 edges. The total distance traveled = 624″ + 72″x2 = 768″ = 64′.

Again, let the spider travel half way around the base to C, then return to B. Ascend to S; descend to C. Finish the perimeter of the base to A. Return to D and ascend to S and complete the circuit. Total distance traveled = 768″ = 64′.

This shortest journey can be made in sixteen ways as follows:

SBADCBCSADS
SBADCBCSDAS
SBCDABCSADS
SBCDABCSDAS

SCBADCBSADS
SCBADCBSDAS
SCDABCBSADS
SCDABCBSDAS

SADCBADSCBS
BCS
SABCDADSCBS
BCS

SDCBADASCBS
BCS
SDABCDASCBS
BCS

No. 115. Proposed by G. W. Wishard, Norwood, Ohio.

Prove that every odd square in the octonary system (scale of eight) ends in 1, and if this 1 be cut off, the remaining part is a triangular number.

Solution by the proposer.

$$(2n+1)^2 = 4n^2+4n+1 = 4n(n+1)+1 = 8k+1$$

Therefore, every odd square in the octonary system ends in 1. Now if the final 1 be cut off we have, $4n(n+1)$ and if the last position is removed, that is, if the number is divided by eight, we have

$$\frac{4n(n+1)}{8} = \frac{n(n+1)}{2} = 1+2+3+\ldots+n$$

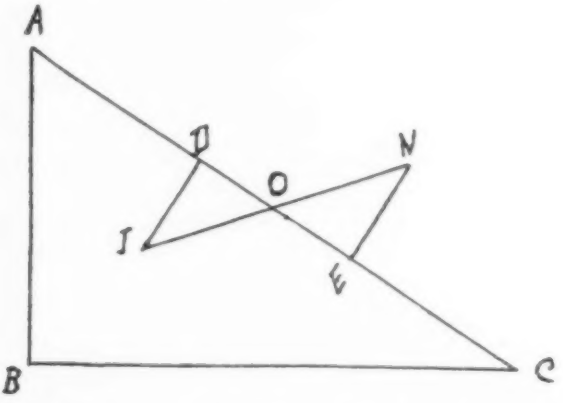
Therefore, the remaining part will be a triangular number in the same system.

Note: This important property of the odd squares is overlooked by the advocates of the octonary system.

No. 116. Proposed by Walter B. Clarke, San Jose, Cal.

I is the incenter, J and K are any two of the excenters of a right triangle. Let perpendiculars from J and K to the nearest sides of the triangle intersect at N. Show that I and N are equidistant from the hypotenuse.

Solved by *Henry Schroeder*, Louisiana Polytechnic Institute.



Designate D foot of perpendicular from I on hypotenuse, and E foot of perpendicular from N on hypotenuse.

In any right triangle the circumcenter, O, lies on mid point of hypotenuse. I, O, N are collinear and IO = ON. (N is circumcenter, O is nine point center, and I is orthocenter of triangle formed by excenters.)

AD = CE. Hence EO = OD. (Altshiller-Court College Geometry. Page 75.) Triangles EON and IDO are congruent and NE = ID.

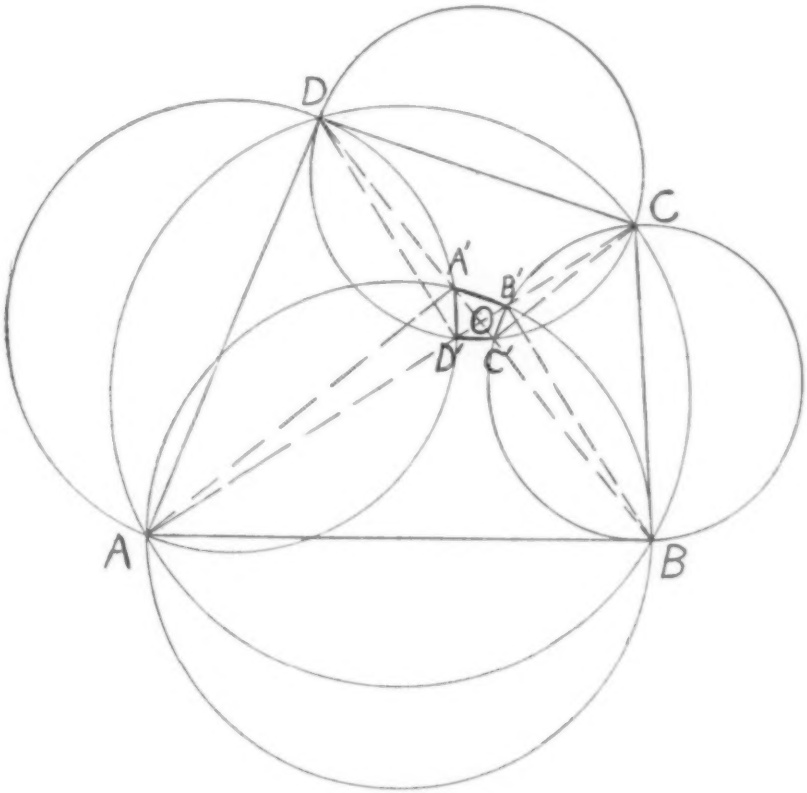
Q. E. D.

Also solved by *A. C. Briggs, Karleton W. Crain, R. A. Miller,* and a remark is given by *Davis P. Richardson.*

No. 117. Proposed by Walter B. Clarke.

Prove that four circles whose diameters are sides of a cyclic quadrilateral intersect in four concyclic points.

Solved by *Karleton W. Crain,* Purdue.



Let ABCD be the given quadrilateral. Draw the four circles whose diameters are the sides of ABCD. Let A′ be the other intersection of the two circles drawn through A; similarly determine the points B′, C′, and D′. Draw AA′, BB′, CC′, and DD′. Let O be the intersection of the diagonals of A′B′C′D′.

Now A,D′,C are collinear, for $\angle AD'D = \angle DD'C = 90°$. A,B′,C are collinear, for $\angle AB'B = \angle CB'B = 90°$. Therefore, A,D′,B′,C are

all collinear. Similarly D,A′,B are collinear for $\angle AA'D = \angle BA'A = 90°$. Also D,C′,B are collinear for $\angle DC'C = \angle CC'B = 90°$. Therefore, D,A′,C′,B are all collinear.

The triangles AA′O, BB′O, CC′O, DD′O are similar right triangles with their right angles at A′,B′,C′,D′ respectively. For angles AA′B, BB′C, CC′D, and DD′A are all angles inscribed in semicircles and the angles at O of these triangles are all equal. Therefore,

$$A'O : B'O : C'O : D'O = AO : BO : CO : DO,$$

and A′B′C′D′ is inversely similar to ABCD. As ABCD is a cyclic quadrilateral, A′B′C′D′ is also cyclic.

Also solved by *A. C. Briggs*.

PROBLEMS FOR SOLUTION

No. 124. Proposed by Nathan Altshiller-Court, University of Oklahoma.

The point of contact of each face of a tetrahedron with the escribed sphere relative to this face* is joined to the mid-point of the altitude relative to the face considered. The four lines thus obtained are concurrent.

No. 125. Proposed by Nathan Altshiller-Court, University of Oklahoma.

If an arbitrary plane through the centroids G of a tetrahedron DABC† cuts the edges DA, DB, DC, in the points P, Q, R, we have, both in magnitude and in sign,

$$\frac{AP}{PD} + \frac{BQ}{QD} + \frac{CR}{RD} = 1$$

No. 126. Proposed by Brother Northbert, University of Portland, Portland, Oregon.

If the line L_1 is the polar of the point P_1 in respect to a family of circles, then the circles have a common radical axis which is the perpendicular bisector of the perpendicular line-segment joining the pole with the polar.

*Nathan Altshiller-Court, Modern Pure Solid Geometry, page 76.
†Nathan Altshiller-Court, Modern Pure Solid Geometry, page 52.

No. 127. Proposed by Richard A. Miller, University of Iowa.

Using the notation, I for incenter, O as circumcenter, K as Nagel point (the intersection of the perpendiculars from the excenters to the sides), T as the centroid of the triangle formed by the excenters I′, I″, I‴; Prove T,I,O,K are collinear, and IO : OT : TK = 3 : 1 : 2.

No. 128. Proposed by Henry Schroeder, Louisiana Polytechnic Institute.

What sum of money is required at 6% nominal, compounded continuously, to purchase an annuity of $10.00 per day for twenty years. (Would like to have a formula developed by Use of Differential Equations.)

No. 129. Proposed by Walter B. Clarke, San Jose, Cal.

In the triangle ABC, I is the incenter, O is the circumcenter, N′ is the center of the nine-point circle, C′ is the midpoint of AB and M is the concurrent point of the lines from the mid-points of the sides to the point half way around the perimeter. Show that

$$MN' : MC' :: IO : IC$$

LATE SOLUTIONS

No. 106 and No. 108, by *A. C. Briggs*.

Book Reviews

Edited by
P. K. SMITH

Field Work in Mathematics. By Carl N. Shuster and Fred L. Bedford. American Book Company, 1935. New York, N. Y. VII +149 pages.

The introduction includes the following statement from Lord Kelvin: "When you can measure what you are speaking about and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express in numbers, your knowledge is of a meager and unsatisfactory kind." The spirit of this elementary text is expressed in this statement from Lord Kelvin. The text is an attempt on the part of the authors to generate interest by allowing the student to secure the numerical quantities with which he is to deal by actual measurement in the field.

The text is better adapted for material to be used supplementary to courses in arithmetic, geometry, algebra, trigonometry, geography and physics.

In this text there are fifteen chapters of subject matter, an appendix, a chapter containing review tests, a set of tables of logarithms and values of the trigonometric functions, and an index.

Chapter I is devoted to a discussion of the use of instruments in mathematics. The history of a few early instruments is given in Chapter II. Chapters III and IV are devoted to scale drawing and approximate numbers, respectively. Chapter V is a fourteen page treatment of the use of the slide rule. Examples of the use of the slide rule in trigonometry, chemistry and physics are given. Linear measurements with the chain and tape are discussed in Chapter VI. The angle mirror, hypsometer and clinometer, and their uses are described in chapters VII and VIII. The use of simple instruments valuable in Scout work is given in chapter IX. A brief discussion of the plane table is given in chapter X. The vernier is explained in chapter XI. Chapter XII is devoted to an eighteen page treatment of the sextant and its uses. In chapter XIII the determination of the meridian is taken up. A thirty-four page treatment of the transit and and its uses is given in chapter XIV. The methods of finding area is covered in the last chapter of subject matter proper.

In the preface the following uses of this text are suggested:

(1) For teacher-training institutions wishing to give a course similar to that in which this material is developed.
(2) For teachers of science and mathematics and Scout leaders who want to familiarize themselves with some practical applications of mathematics.
(3) As an introduction to a college engineering course.
(4) As a source of supplementary material for enrichment of high-school courses in science and mathematics.
(5) As an interesting course in high school mathematics to follow elementary algebra.
(6) As a course for teachers of geography who wish to learn how maps are made.
(7) As a source book for practical amateur engineers.

This text is very interesting. The material is not sufficiently developed to use as a text independently. The subject matter of trigonometry in the text is meager even though exercises in measurements call for the use of oblique triangles.

The writer feels that the material, if properly used along with the instruments (provided the money could be found for buying the instruments), would add greatly to the interest in high school and college trigonometry classes. The appearance of the book is pleasing and the figures are splendid. The text is worthy of a close scrutiny by anyone interested in a course of the nature described.

P. K. SMITH,
Louisiana Polytechnic Institute.

Analytic Geometry. By Charles H. Sisam. Henry Holt and Company, New York, 1936, xix+310 pp.

In writing this elementary textbook the author says in the preface that he has constantly kept in mind the actual needs and desires of the teachers of analytic geometry. The reviewer thinks the author has thus produced the sort of textbook the students really need and desire to have in studying analytic geometry.

The author recognizes that the course in analytic geometry has several major objectives. He says, "It should follow in a natural way from the student's previous work in mathematics, which it is expected to unify; it must acquaint the student with the methods, the spirit, and the essential facts of analytic geometry; and it should stress the

particular types of reasoning the student will encounter most frequently in later work."

The first objective has largely been effected by the arrangement of the chapters and the other objectives by the well chosen material put therein and the simple and clear expositions. In the introduction of seven pages are gathered together, for reference, definitions, theorems, and formulas from algebra, geometry, and trigonometry. In chapter I coordinates and graphs are treated very much as the students should be familiar with in the study of simultaneous equations, linear and quadratic. In chapter II the fundamental definitions and theorems are clearly set forth in words, symbols, and figures. The directed line segment from A to B on a straight line is denoted by $\overline{AB}$ and the undirected line segment by AB. Hence $\overline{AB}=AB$ and $\overline{BA}=-AB$. The use of directed and undirected line segments in deriving formulas and equations is very clear.

Chapter III is devoted to the line, chapter IV to the circle, and chapter V to the conic sections. Though the three conic sections are in a single chapter, the parabola, the ellipse, and the hyperbola are treated under separate headings. The standard equation of the ellipse is developed on page 95 from the definition as a locus of a point that moves so that the sum of its undirected distances from two fixed points is equal to a constant, and on page 101 from the definition as a locus of a point that moves in such a way that the ratio of its undirected distance from the focus $F(ae,0)$ to its undirected distance from the directrix $x-a/e=0$ is equal to e, the eccentricity. The two developments of the standard form of the equation of the hyperbola are similarly given on pages 106 and 112. In chapter VI transformation of coordinates is treated under translation of axes and rotation of axes, the equailateral hyperbola being placed there. The applications of this chapter come mainly in the next chapter on the general equation of the second degree, all of which is concisely done in ten pages.

Higher plane curves, algebraic and transcendental, are simply treated in chapter VIII, while parametric equations are well put in the separate chapter just following.

Polar coordinates, being introduced in the second chapter, are employed in the separate chapters dealing with the line, the circle, and the conic sections. Other useful curves in polar coordinates are brought together in chapter X. Tangents, normals, diameters, poles, and polars are adequately treated for a first course in chapter XI. The tangent line to a curve being defined as the limiting position of a secant line, the slope of the tangent line is worked out as the limit of the ratio,

$\Delta y/\Delta x$, no treatment of derivatives being given. In empirical equations the common types are chosen and are very simply treated.

Solid analytic geometry is given in sixty pages in three chapters, fundamental definitions and theorems, planes and lines, and types of surfaces and curves.

The textbook is well supplied with exercises, more than a thousand, printed not in small type but in a type nearly the size of the type of the text. Miscellaneous and selected exercises are collected in separate groups at the ends of the chapters on definitions and theorems, line, circle, conic sections, general equation of second degree, and the first two of the chapters of solid analytic geometry. The selected exercises are to stimulate and challenge distinctly superior students. Answers have been given to about one-third of the exercises, none for the selected exercises. Among the exercises is a very good proportion of problems of practical nature.

In realizing that well drawn figures aid materially in solving problems, the author has supplied the book with 171 figures, more than one-third of them belonging to worked out examples. The figures are well drawn and effectively placed on the pages. Tables of four-place logarithms and natural trigonometric functions are included as well as a three-place table of square roots.

WILSON L. MISER,
Vanderbilt University.

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